Eco 362: Economic Growth
Fall 2013
Solutions for Problem Set 1

Note: all Weil question numbers refer to the 3rd edition textbook.

Question 1: Weil Chapter 1, Q 6

Solution: In order to calculate the year in which income per capita in the United States was equal to income per capita in Sri Lanka, we need to find t, the number of years that passed between the year 2009 and the year U.S. income per capita equaled that of 2009 Sri Lanka income per capita. Equating income per capita of Sri Lanka in year 2009 to income per capita of the United States in year $2009 - t$, we now write an equation for the United States as

$$Y_{U.S.,2009-t} * (1+g)^t = Y_{U.S.,2009}$$

Since

$$\begin{aligned} Y_{U.S.,2009-t} &= Y_{SriLanka,2009} = \$4,034, \\ Y_{U.S.,2009} &= \$41,099, and \\ g &= 0.018, \end{aligned}$$

we then substitute in these values and solve for t.

$$\begin{aligned} (\$4,034 * (1+0.018)^t &= \$41,099 \\ (1+0.018)^t &= \$41,099/\$4,034 \end{aligned}$$

One can solve for t by simply trying out different values on a calculator. Alternatively, taking the natural log of both sides, and noting that

$$ln(x^y) = yln(x)$$

we get

$$\begin{aligned} tln(1+0.018) &= ln(\$41,099/\$4,034) \\ t &= 130.11. \end{aligned}$$

That is, 130.11 years ago, the income per capita of the United States equaled that of Sri Lanka's income in the year 2009. This year was roughly$2009-t$, i.e., the year 1879.

Question 2: Weil Chapter 3, Q 2

Solution: The information given to us is that the production function is $y = k^{1/2} i.e. A = 1, \alpha = 1/2$ for our general production function we used in class. We are also given that , $\gamma = .5, \delta = 0.05, n = 0$

The steady state k and y is given by

$$\begin{aligned} k^{SS} &= \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\alpha}} \\ y^{SS} &= A\left(k^{SS}\right)^{\alpha} \end{aligned}$$

after substituting in the parameters, we get

$$\begin{aligned} k^{SS} &= \left(\frac{0.5}{0.05}\right)^{2} = 100 \\ y^{SS} &= (100)^{1/2} = 10 \end{aligned}$$

Since we are given that the economy is at $k = 400$ which implies $y = 20$, we are currently above steady state.

Question 3: Weil Chapter 4, Q7

Solution: Country A and B are identical in every respect but for their population growth rates. This implies that their respective steady states are not equal. The ratio of their steady state incomes is given by

$$\begin{aligned} \frac{y_A^{SS}}{y_B^{SS}} &= \frac{A\left(\frac{\gamma A}{n_A+\delta}\right)^{\frac{\alpha}{1-\alpha}}}{A\left(\frac{\gamma A}{n_B+\delta}\right)^{\frac{\alpha}{1-\alpha}}} \\ &= \left(\frac{n_B+\delta}{n_A+\delta}\right)^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

Utilizing the fact that $n_A > n_B$ and we can conclude that $y_A^{SS} < y_B^{SS}$. Because both countries currently have the same income per capita, we know that Country B is farther away from its respective steady-state level. Therefore, Country B must have a higher growth rate of output per worker than does Country A as we know that the farther an economy is from its steady state, the faster it grows.

Question 4: Consider the Solow model where the production function is given by $Y_t = AK_t^{\alpha} L_t^{1-\alpha}, 0 < \alpha < 1$

a) Derive the equation for the evolution of $k_t = \frac{K_t}{L_t}$

b) Discuss the effects on the economy of a **one time, permanent** increase in the **population growth rate**. Discuss the effects graphically and in words.

Answer:

a) Assumptions:

- The production function is given by $Y_t = AK_t^{\alpha} L_t^{1-\alpha}, 0 < \alpha < 1$

- $I_t = \gamma Y_t$ (for γ constant)
- Population growth rate $= n$ (constant) $L_{t+1} = (1+n) L_t$

We start with the capital accumulation equation

$$
\begin{aligned}
K_{t+1} &= (1-\delta) K_t + I_t \\
K_{t+1} &= (1-\delta) K_t + \gamma Y_t \\
K_{t+1} &= (1-\delta) K_t + \gamma A K_t^{\alpha} L_t^{1-\alpha} \\
\frac{K_{t+1}}{L_{t+1}} &= \frac{(1-\delta) K_t + \gamma A K_t^{\alpha} L_t^{1-\alpha}}{L_{t+1}} \\
k_{t+1} &= \frac{(1-\delta) K_t + \gamma A K_t^{\alpha} L_t^{1-\alpha}}{(1+n) L_t} \\
k_{t+1} &= \frac{1}{1+n}\left[(1-\delta)\frac{K_t}{L_t} + \gamma A \frac{K_t^{\alpha} L_t^{1-\alpha}}{L_t}\right] \\
k_{t+1} &= \frac{1}{1+n}\left[(1-\delta) k_t + \gamma A k_t^{\alpha}\right]
\end{aligned}
$$

Note: Make sure you know how to derive the equations we use here because the derivations can be asked on the exams. Also make sure you know what the notation (i.e.$\delta, \gamma, n, \alpha$) stands for.

b) The economy is moving from a low population growth rate (n_{old}) to a higher population growth rate $(n_{new} > n_{old})$

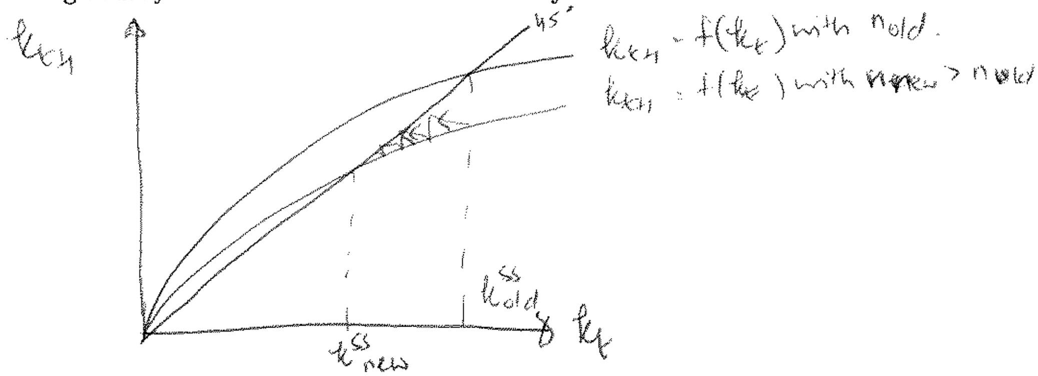
Effects on k :

First let's start by discussing the effects on k i.e. the capital to labor ratio, $\left(k_t = \frac{K_t}{L_t}\right)$. The equation that governs the evolution of k is given by

$$k_{t+1} = \frac{(1-\delta) k_t + \gamma k_t^{\alpha}}{1+n}$$

Consider a country starting out in steady state $\left(i.e. \text{ at } k_{old}^{SS}\right)$. An increase in the population growth rate will shift the line that shows the relationship between k_{t+1} and k_t downwards. That is, for every k_t the economy will have a lower k_{t+1} with the higher population growth rate (n_{new}) than it did with the old population growth rate (n_{old}). The reason is that the investment behavior remains the same, but now the country has to share the capital among a larger workforce. This implies that the steady state of the economy is now lower $\left(k_{new}^{SS} < k_{old}^{SS}\right)$. The economy is currently at higher k relative to its new steady state and so its

k will gradually shrink till it reaches the new steady state



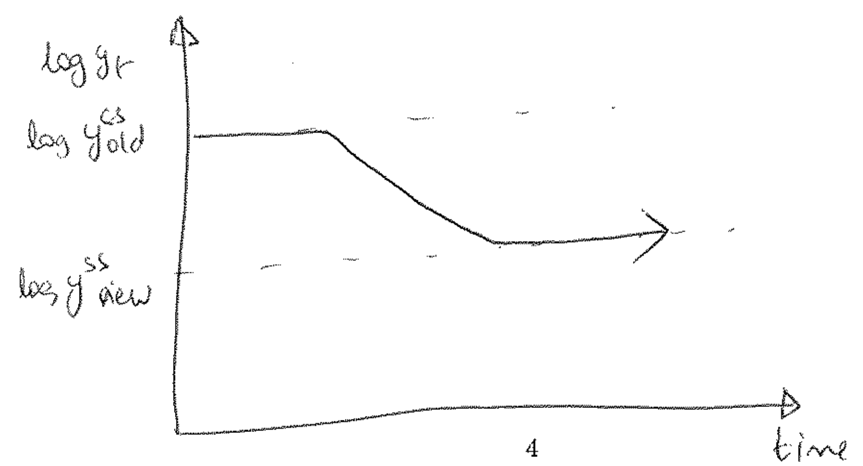
The long run effect on k is level effect of moving from a higher to a lower steady state level. The long run growth rate of k is still zero. In the short run it has negative growth till it reaches its steady state.

Effects on y :

Per-capita income (y) is a function of k and so mirrors its behavior.

$$\begin{aligned}
y &= \frac{Y_t}{L_t} \\
&= \frac{AK_t^{\alpha} L_t^{1-\alpha}}{L_t} \\
&= AK_t^{\alpha} L_t^{-\alpha} \\
&= Ak_t^{\alpha}
\end{aligned}$$

It starts out at its steady state $\left(y_{old}^{SS} = A\left(k_{old}^{SS}\right)^{\alpha}\right)$. When the population growth rate increases, k gradually decreases till it gets to its new steady state $y_{new}^{SS} = A\left(k_{new}^{SS}\right)^{\alpha}$. In the same way, y steady decreases till it gets to its new, lower, steady state $\left(y_{new}^{SS}\right)$. The long run effect on y is level effect of moving from a higher to a lower steady state level. The long run growth rate of y is still zero. In the short run it has negative growth till it reaches its steady state.



Effects on Y :

While the country is in its old steady state, the long run growth rate of Y is the same as the population growth rate $(i.e. n_{old})$. The easiest way to think through the effects of a change in n on Y is to remember the definition of per-capita income $\left(y = \frac{Y}{L}\right)$. We have been given that the growth rate of L increases from n_{old} to a higher n_{new}. So the denominator grows at a faster rate compared to before the change. We have also seen that y decreases during the transition to its new lower steady state and once it reaches the steady state it stops growing. From these observations we can deduce that the growth rate of Y will gradually increase from it old steady state growth rate of n_{old} to its new higher steady state growth rate of n_{new}. During the transition we will see an accompanying decrease in y (the numerator grows slower than the denominator) and once Y reaches its new steady state growth rate y remains constant (numerator and denominator grow at the same rate).Also, since we know that in the new steady state $\frac{K}{L}$ is constant we know that the growth rate of the capital stock (K) is the same as the new higher growth rate of L. Remember the intuition for why Y grows at the growth rate of L comes from the constant returns to scale property of the production function.